Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Silver Level S2

Time: 1 hour 30 minutes

Materials required for examination papers	**Items included with question**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C4), the paper reference (6666), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

Suggested grade boundaries for this paper:

A*	A	B	C	D	E
66	58	51	45	39	33

1.

$$\frac{9x^2}{(x-1)^2(2x+1)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(2x+1)}.$$

Find the values of the constants A, B and C.

(4)

June 2009

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2. (*a*) Use integration by parts to find $\int x \sin 3x \;\; \mathrm{d}x$.

(3)

(*b*) Using your answer to part (*a*), find $\int x^2 \cos 3x \;\; \mathrm{d}x$.

(3)

January 2012

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3. A curve C has equation

$$2^x + y^2 = 2xy.$$

Find the exact value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point on C with coordinates (3, 2).

(7)

June 2010

4.

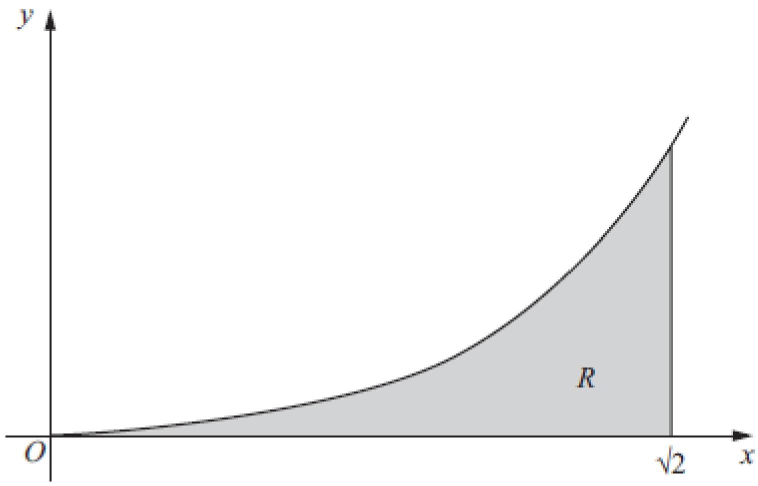

Figure 2

Figure 2 shows a sketch of the curve with equation $y = x^3 \ln (x^2 + 2)$, $x \geq 0$.

The finite region R, shown shaded in Figure 2, is bounded by the curve, the x-axis and the line $x = \sqrt{2}$.

The table below shows corresponding values of x and y for $y = x^3 \ln (x^2 + 2)$.

x	0	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{3\sqrt{2}}{4}$	$\sqrt{2}$
y	0		0.3240		3.9210

(a) Complete the table above giving the missing values of y to 4 decimal places.

(2)

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R, giving your answer to 2 decimal places.

(3)

(c) Use the substitution $u = x^2 + 2$ to show that the area of R is

$$\frac{1}{2}\int_2^4 (u - 2)\ln u \ \mathrm{d}u\,.$$

(4)

(d) Hence, or otherwise, find the exact area of R.

(6)

June 2011

5.

$$\frac{2x^2+5x-10}{(x-1)(x+2)} \equiv A + \frac{B}{x-1} + \frac{C}{x+2}.$$

(a) Find the values of the constants A, B and C.

(4)

(b) Hence, or otherwise, expand $\frac{2x^2+5x-10}{(x-1)(x+2)}$ in ascending powers of x, as far as the term in x^2. Give each coefficient as a simplified fraction.

(7)

June 20108

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6. The area A of a circle is increasing at a constant rate of 1.5 cm^2 s^{-1}. Find, to 3 significant figures, the rate at which the radius r of the circle is increasing when the area of the circle is 2 cm^2.

(5)

January 2010

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7. Relative to a fixed origin O, the point A has position vector $(2\mathbf{i} - \mathbf{j} + 5\mathbf{k})$,

the point B has position vector $(5\mathbf{i} + 2\mathbf{j} + 10\mathbf{k})$,

and the point D has position vector $(-\mathbf{i} + \mathbf{j} + 4\mathbf{k})$.

The line l passes through the points A and B.

(a) Find the vector $\overrightarrow{AB}$.

(2)

(b) Find a vector equation for the line l.

(2)

(c) Show that the size of the angle BAD is 109°, to the nearest degree.

(4)

The points A, B and D, together with a point C, are the vertices of the parallelogram $ABCD$, where $\overrightarrow{AB} = \overrightarrow{DC}$.

(d) Find the position vector of C.

(2)

(e) Find the area of the parallelogram $ABCD$, giving your answer to 3 significant figures.

(3)

(f) Find the shortest distance from the point D to the line l, giving your answer to 3 significant figures.

(2)

January 2012

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8. In an experiment testing solid rocket fuel, some fuel is burned and the waste products are collected. Throughout the experiment the sum of the masses of the unburned fuel and waste products remains constant.

Let x be the mass of waste products, in kg, at time t minutes after the start of the experiment. It is known that at time t minutes, the rate of increase of the mass of waste products, in kg per minute, is k times the mass of unburned fuel remaining, where k is a positive constant.

The differential equation connecting x and t may be written in the form

$$\frac{\mathrm{d}x}{\mathrm{d}t} = k(M - x), \text{ where } M \text{ is a constant.}$$

(a) Explain, in the context of the problem, what $\frac{\mathrm{d}x}{\mathrm{d}t}$ and M represent.

(2)

Given that initially the mass of waste products is zero,

(b) solve the differential equation, expressing x in terms of k, M and t.

(6)

Given also that $x = \frac{1}{2}M$ when $t = \ln 4$,

(c) find the value of x when $t = \ln 9$, expressing x in terms of M, in its simplest form.

(4)

June 2013 (R)

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme		Marks
1.	$9x^2 = A(x-1)(2x+1) + B(2x+1) + C(x-1)^2$		B1
	$x \to 1$ $\quad 9 = 3B \Rightarrow B = 3$		M1
	$x \to -\frac{1}{2}$ $\quad \frac{9}{4} = \left(-\frac{3}{2}\right)^2 C \Rightarrow C = 1$	Any two of A, B, C	A1
	x^2 terms $\quad 9 = 2A + C \Rightarrow A = 4$	All three correct	A1 **(4)** **[4]**

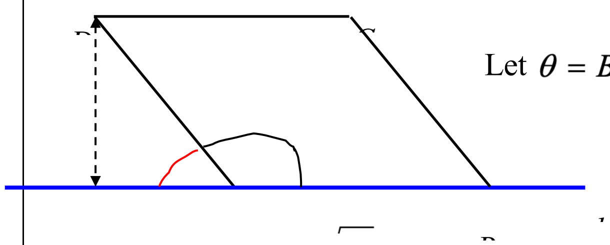
2. (a)	$\int x\sin 3x \,\mathrm{d}x = -\frac{1}{3}x\cos 3x - \int -\frac{1}{3}\cos 3x \,\{\mathrm{d}x\}$		M1 A1
	$= -\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x \,\{+c\}$		A1
			[3]
(b)	$\int x^2\cos 3x \,\mathrm{d}x = \frac{1}{3}x^2\sin 3x - \int \frac{2}{3}x\sin 3x \,\{\mathrm{d}x\}$		M1 A1
	$= \frac{1}{3}x^2\sin 3x - \frac{2}{3}\left(-\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x\right) \,\{+c\}$		A1 **isw**
	$\left\{= \frac{1}{3}x^2\sin 3x + \frac{2}{9}x\cos 3x - \frac{2}{27}\sin 3x \,\{+c\}\right\}$	*Ignore subsequent working*	**[3]**
			(6 marks)

3.	$\frac{\mathrm{d}}{\mathrm{d}x}(2^x) = \ln 2.2^x$		B1
	$\ln 2.2^x + 2y\frac{\mathrm{d}y}{\mathrm{d}x} = 2y + 2x\frac{\mathrm{d}y}{\mathrm{d}x}$		M1 A1= A1
	Substituting $(3, 2)$		
	$8\ln 2 + 4\frac{\mathrm{d}y}{\mathrm{d}x} = 4 + 6\frac{\mathrm{d}y}{\mathrm{d}x}$		M1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 4\ln 2 - 2$	Accept exact equivalents	M1 A1 **(7)** **[7]**

Question Number	Scheme	Marks
4.	(a) $0.0333,\ 1.3596$ awrt $0.0333,\ 1.3596$	B1 B1 **(2)**
	(b) $\text{Area}(R) \approx \frac{1}{2} \times \frac{\sqrt{2}}{4}[\ ...]$	B1
	$\approx\ ...\ \left[0 + 2(0.0333 + 0.3240 + 1.3596) + 3.9210\right]$	M1
	≈ 1.30 Accept 1.3	A1 **(3)**
	(c) $u = x^2 + 2 \Rightarrow \frac{du}{dx} = 2x$	B1
	$\text{Area}(R) = \int_0^{\sqrt{2}} x^3 \ln(x^2+2)\,dx$	B1
	$\int x^3 \ln(x^2+2)\,dx = \int x^2 \ln(x^2+2)\,x\,dx = \int (u-2)(\ln u)\tfrac{1}{2}\,du$	M1
	Hence $\text{Area}(R) = \frac{1}{2}\int_2^4 (u-2)\ln u\,du$ * cso	A1 **(4)**
	(d) $\int (u-2)\ln u\,du = \left(\frac{u^2}{2} - 2u\right)\ln u - \int \left(\frac{u^2}{2} - 2u\right)\frac{1}{u}\,du$	M1 A1
	$= \left(\frac{u^2}{2} - 2u\right)\ln u - \int \left(\frac{u}{2} - 2\right)du$	
	$= \left(\frac{u^2}{2} - 2u\right)\ln u - \left(\frac{u^2}{4} - 2u\right) \quad (+C)$	M1 A1
	$\text{Area}(R) = \frac{1}{2}\left[\left(\frac{u^2}{2} - 2u\right)\ln u - \left(\frac{u^2}{4} - 2u\right)\right]_2^4$	
	$= \tfrac{1}{2}\left[(8-8)\ln 4 - 4 + 8 - \left((2-4)\ln 2 - 1 + 4\right)\right]$	M1
	$= \tfrac{1}{2}(2\ln 2 + 1)$ $\ln 2 + \tfrac{1}{2}$	A1 **(6)**
		[15]

5.	(a) $A=2$	B1
	$2x^2+5x-10=A(x-1)(x+2)+B(x+2)+C(x-1)$	
	$x\to1$ $\quad -3=3B \Rightarrow B=-1$	M1 A1
	$x\to-2$ $\quad -12=-3C \Rightarrow C=4$	A1 **(4)**
	(b) $\dfrac{2x^2+5x-10}{(x-1)(x+2)}=2+(1-x)^{-1}+2\left(1+\dfrac{x}{2}\right)^{-1}$	M1
	$(1-x)^{-1}=1+x+x^2+\ \ldots$	B1
	$\left(1+\dfrac{x}{2}\right)^{-1}=1-\dfrac{x}{2}+\dfrac{x^2}{4}+\ \ldots$	B1
	$\dfrac{2x^2+5x-10}{(x-1)(x+2)}=(2+1+2)+(1-1)x+\left(1+\dfrac{1}{2}\right)x^2+\ \ldots$	M1
	$=5+\ \ldots$ ft their $A-B+\frac{1}{2}C$	A1 ft
	$=\ \ldots\ +\dfrac{3}{2}x^2+\ \ldots$ $0x$ stated or implied	A1 A1 **(7)**
		[11]

Question Number	Scheme	Marks
Q6	$\dfrac{\mathrm{d}A}{\mathrm{d}t}=1.5$	B1
	$A=\pi r^2 \Rightarrow \dfrac{\mathrm{d}A}{\mathrm{d}r}=2\pi r$	B1
	When $A=2$	
	$2=\pi r^2 \Rightarrow r=\sqrt{\dfrac{2}{\pi}}\ (=0.797\,884\ \ldots)$	M1
	$\dfrac{\mathrm{d}A}{\mathrm{d}t}=\dfrac{\mathrm{d}A}{\mathrm{d}r}\times\dfrac{\mathrm{d}r}{\mathrm{d}t}$	
	$1.5=2\pi r\dfrac{\mathrm{d}r}{\mathrm{d}t}$	M1
	$\dfrac{\mathrm{d}r}{\mathrm{d}t}=\dfrac{1.5}{2\pi\sqrt{\frac{2}{\pi}}}\approx0.299$ awrt 0.299	A1
		[5]

Question Number	Scheme		Marks
7.	$\overrightarrow{OA} = 2\mathbf{i} - \mathbf{j} + 5\mathbf{k}$, $\overrightarrow{OB} = 5\mathbf{i} + 2\mathbf{j} + 10\mathbf{k}$, $\left\{\overrightarrow{OC} = 2\mathbf{i} + 4\mathbf{j} + 9\mathbf{k}\right\}$ & $\overrightarrow{OD} = -\mathbf{i} + \mathbf{j} + 4\mathbf{k}$		
(a)	$\overrightarrow{AB} = = \pm\left((5\mathbf{i} + 2\mathbf{j} + 10\mathbf{k}) - (2\mathbf{i} - \mathbf{j} + 5\mathbf{k})\right); = 3\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$		M1; A1 **[2]**
(b)	$l: \mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix} 5 \\ 2 \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix}$		M1 A1ft **[2]**
	 Let $\theta = B\hat{A}D$	Let d be the shortest distance from C to l.	
(c)	$\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA} = \begin{pmatrix} -1 \\ 1 \\ 4 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} = \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix}$ or $\overrightarrow{DA} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$		M1
	$\cos\theta = \dfrac{\overrightarrow{AB} \bullet \overrightarrow{AD}}{\lvert\overrightarrow{AB}\rvert . \lvert\overrightarrow{AD}\rvert} = \dfrac{\begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix} \bullet \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix}}{\sqrt{(3)^2 + (3)^2 + (5)^2} . \sqrt{(-3)^2 + (2)^2 + (-1)^2}}$	Applies dot product formula between their $\left(\overrightarrow{AB} \text{ or } \overrightarrow{BA}\right)$ and their $\left(\overrightarrow{AD} \text{ or } \overrightarrow{DA}\right)$.	M1
	$\cos\theta = \pm\left(\dfrac{-9 + 6 - 5}{\sqrt{(3)^2 + (3)^2 + (5)^2} . \sqrt{(-3)^2 + (2)^2 + (-1)^2}}\right)$	Correct followed through expression or **equation**.	A1 $\sqrt{\ }$
	$\cos\theta = \dfrac{-8}{\sqrt{43} . \sqrt{14}} \Rightarrow \theta = 109.029544... = 109$ (nearest °)	awrt 109	A1 **cso AG** **[4]**
(d)	$\overrightarrow{OC} = \overrightarrow{OD} + \overrightarrow{DC} = \overrightarrow{OD} + \overrightarrow{AB} = (-\mathbf{i} + \mathbf{j} + 4\mathbf{k}) + (3\mathbf{i} + 3\mathbf{j} + 5\mathbf{k})$ $\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC} = \overrightarrow{OB} + \overrightarrow{AD} = (5\mathbf{i} + 2\mathbf{j} + 10\mathbf{k}) + (-3\mathbf{i} + 2\mathbf{j} - \mathbf{k})$		M1
	So, $\overrightarrow{OC} = 2\mathbf{i} + 4\mathbf{j} + 9\mathbf{k}$		A1 **[2]**
(e)	Area $ABCD = \left(\tfrac{1}{2}(\sqrt{43})(\sqrt{14})\sin 109^\circ\right); \times 2 = 23.19894905$	awrt 23.2	M1; dM1 A1 **[3]**
(f)	$\dfrac{d}{\sqrt{14}} = \sin 71$ or $\sqrt{43}\, d = 23.19894905...$		M1
	$\therefore\ d = \sqrt{14}\sin 71^\circ = 3.537806563...$	awrt 3.54	A1 **[2]** **(15 marks)**

Question Number	Scheme	Marks
8.	$\frac{dx}{dt} = k(M - x)$, where M is a constant	
(a)	$\frac{dx}{dt}$ is the <u>rate of increase</u> of the <u>mass of waste</u> products. — Any one correct explanation.	B1
	M is the <u>total mass</u> of <u>unburned fuel</u> and <u>waste fuel</u> (or the <u>initial mass</u> of <u>unburned fuel</u>) — Both explanations are correct.	B1
		(2)
(b)	$\int \frac{1}{M-x}\,dx = \int k\,dt$ or $\int \frac{1}{k(M-x)}\,dx = \int dt$	B1
	$-\ln(M-x) = kt \ \{+c\}$ or $-\frac{1}{k}\ln(M-x) = t\ \{+c\}$	M1 A1
	$\{t=0,\ x=0 \Rightarrow\}\ -\ln(M-0) = k(0) + c$	M1
	$c = -\ln M \Rightarrow -\ln(M-x) = kt - \ln M$	
	then either... $-kt = \ln(M-x) - \ln M$; $-kt = \ln\left(\frac{M-x}{M}\right)$; $e^{-kt} = \frac{M-x}{M}$; $Me^{-kt} = M - x$ ***or...*** $kt = \ln M - \ln(M-x)$; $kt = \ln\left(\frac{M}{M-x}\right)$; $e^{kt} = \frac{M}{M-x}$; $(M-x)e^{kt} = M$; $M - x = Me^{-kt}$	ddM1
	leading to $x = M - Me^{-kt}$ or $x = M(1 - e^{-kt})$ oe	A1 * **cso**
		(6)
(c)	$\left\{x = \frac{1}{2}M,\ t = \ln 4 \Rightarrow\right\}\ \frac{1}{2}M = M(1 - e^{-k\ln 4})$	M1
	$\Rightarrow \frac{1}{2} = 1 - e^{-k\ln 4} \Rightarrow e^{-k\ln 4} = \frac{1}{2} \Rightarrow -k\ln 4 = -\ln 2$	
	So $k = \frac{1}{2}$	A1
	$x = M\left(1 - e^{-\frac{1}{2}\ln 9}\right)$	dM1
	$x = \frac{2}{3}M$ — $x = \frac{2}{3}M$	A1 **cso**
		(4) **[12]**

Question 1

The majority of candidates gained full marks on this question. Most obtained the identity $9x^2 \equiv A(x-1)(2x+1)+B(2x+1)+C(x-1)^2$ and found B and C by substituting $x=1$ and $x=-\frac{1}{2}$. A significant number of candidates found an incorrect value of C after making the error $\left(-\frac{3}{2}\right)^2=-\frac{9}{4}$. This can arise through the misuse of a calculator. The value of A was usually found either by substituting $x=0$ or equating coefficients of x^2. Relatively few candidates attempted the question by equating all three coefficients to obtain three equations and solving these equations simultaneously. The working for this method is rather complicated and errors were often made.

Question 2

This question was generally well answered with around 50% of the candidature gaining all 6 marks. The majority of candidates were able to apply the integration by parts formula in the correct direction. Some candidates, however, did not assign u and $\frac{\mathrm{d}v}{\mathrm{d}x}$ and then write down their $\frac{\mathrm{d}u}{\mathrm{d}x}$ and v before applying the by parts formula, which meant that if errors were made the method used was not always clear.

In part (a), $\int \sin 3x\,\mathrm{d}x$ caused some problems for a minority of candidates who produced responses such as $\pm\cos 3x$ or $\pm 3\cos 3x$ or $\frac{1}{3}\cos 3x$. After correctly applying the by parts formula, a few candidates then incorrectly wrote down $\frac{1}{3}\int \cos 3x\,\mathrm{d}x$ as $\frac{1}{6}\cos 3x$.

Most candidates who could attempt part (a) were able to make a good start to part (b), by assigning u as x^2 and $\frac{\mathrm{d}v}{\mathrm{d}x}$ as $\cos 3x$, and then correctly apply the integration by parts formula. At this point, when faced with $\frac{2}{3}\int x\sin 3x\,\mathrm{d}x$, some candidates did not make the connection with their answer to part (a) and made little progress. Other candidates independently applied the by parts formula again, with a number of them making a sign error.

Question 3

This question was also well answered and the general principles of implicit differentiation were well understood. By far the commonest source of error was in differentiating 2^x; examples such as 2^x, $2^x \ln x$ and $x2^{x-1}$ were all regularly seen. Those who knew how to differentiate 2^x nearly always completed the question correctly, although a few had difficulty in finding $\frac{\mathrm{d}}{\mathrm{d}x}(2xy)$ correctly. A minority of candidates attempted the question by taking the logs of both sides of the printed equation or a rearrangement of the equation in the form

$2^x = 2xy - y^2$. Correctly done, this leads to quite a neat solution, but, more frequently, errors, such as $\ln\left(2^x + y^2\right) = \ln 2^x + \ln y^2$, were seen.

Question 4

Part (a) was well done and the only error commonly seen in part (b) was using the incorrect width of the trapezium $\frac{\sqrt{2}}{5}$ instead of $\frac{\sqrt{2}}{4}$. A few candidates made errors, often due to a lack of clear bracketing, but great majority completed part (b) correctly and gave their answer to the degree of accuracy specified in the question. Part (c) was well done and the majority were able to find $\frac{\mathrm{d}u}{\mathrm{d}x}$ and make a complete substitution for the variables. The only common error in this part was simply to ignore the limits and to give no justification for the limits becoming 2 and 4. Most recognised that the integral in part (d) required integration by parts and those who used a method involving integrating $(u-2)$ to $\frac{u^2}{2} - 2u$ and differentiating $\ln u$ usually reached the half way stage correctly. The second integration proved more difficult and there were many errors in simplifying the expression $\left(\frac{u^2}{2} - 2u\right)\frac{1}{u}$ before the second integration. The errors often arose from a failure to use the necessary brackets. There were also many subsequent errors in signs and a few candidates omitted the $\frac{1}{2}$ from their integration.

Those who, at the first stage of integration by parts integrated $(u-2)$ to $\frac{(u-2)^2}{2}$, which is, of course, correct, had markedly less success with the second integral than those who integrated to $\frac{u^2}{2} - 2u$.

A few split the integral up into two separate integrals, $\int u \ln u \,\mathrm{d}u$ and $\int \ln u \,\mathrm{d}u$ but the second of these integrals was rarely completed correctly. Those who ignored the hint in the question and attempted to integrate with respect to x were generally unable to deal with $\int \frac{x^5}{x^2+2}\mathrm{d}x$, which arises after integrating by parts once

Question 5

The first part of question 5 was generally well done. Those who had difficulty generally tried to solve sets of relatively complicated simultaneous equations or did long division obtaining an incorrect remainder. A few candidates found B and C correctly but either overlooked finding A or did not know how to find it. Part (b) proved very testing. Nearly all were able to make the connection between the parts but there were many errors in expanding both $(x-1)^{-1}$ and $(2+x)^{-1}$. Few were able to write $(x-1)^{-1}$ as $-(1-x)^{-1}$ and the resulting expansions were incorrect in the majority of cases, both $1+x-x^2$ and $1-x-x^2$ being common.

$\left(2+x\right)^{-1}$ was handled better but the constant $\frac{1}{2}$ in $\frac{1}{2}\left(1+\frac{x}{2}\right)^{-1}$ was frequently incorrect. Most recognised that they should collect together the terms of the two expansions but a few omitted their value of A when collecting the terms.

Question 6

Connected rates of change is a topic which many find difficult. The examiners reported that the responses to this question were of a somewhat higher standard than had been seen in some recent examinations and the majority of candidates attempted to apply the chain rule to the data of the question. Among those who obtained a correct relation, $1.5 = 2\pi r\frac{\mathrm{d}r}{\mathrm{d}t}$ or an equivalent, a common error was to use $r = 2$, instead of using the given $A = 2$ to obtain $r = \sqrt{\frac{2}{\pi}}$. Unexpectedly the use of the incorrect formula for the area of the circle, $A = 2\pi r^2$, was a relatively common error.

Question 7

This question discriminated well across all abilities, with parts (e) and (f) being the most demanding, and those candidates who drew their own diagram being the more successful. About 15% of the candidature was able to gain all 15 marks.

Part (a) was well answered with only a few candidates adding $\overrightarrow{OB}$ to $\overrightarrow{OA}$ instead of applying $\overrightarrow{OB} - \overrightarrow{OA}$. Candidates who failed to answer part (a) correctly usually struggled to gain few if any marks for the remainder of this question.

In part (b), most candidates were able to write down a correct expression for l, but a number of candidates did not form a correct equation by writing either $\mathbf{r}$ = ... and so lost the final accuracy mark. (After some discussion the examiners also accepted l = ..., which is quite common, though non standard.)

In part (c), most candidates were able to take the correct dot product between either $\overrightarrow{AB}$ and $\overrightarrow{AD}$ or $\overrightarrow{BA}$ and $\overrightarrow{DA}$ to obtain the correct answer of 109°. The most common error was to obtain an answer of 71° by incorrectly taking the dot product between either $\overrightarrow{AB}$ and $\overrightarrow{DA}$ or $\overrightarrow{BA}$ and $\overrightarrow{AD}$, and using this answer to obtain an answer of 109° without proper justification. A small minority of candidates applied the cosine rule correctly to achieve the correct answer. A number of candidates struggled with this part and usually took the dot product between non-relevant vectors such as $\overrightarrow{OA}$ and $\overrightarrow{OB}$ or $\overrightarrow{AB}$ and $\overrightarrow{BD}$.

In part (d), a significant number of candidates were able to obtain the correct position vector of $\overrightarrow{OC}$ = 2$\mathbf{i}$ + 4$\mathbf{j}$ + 9$\mathbf{k}$ by adding either $\overrightarrow{OD}$ to $\overrightarrow{AB}$ or $\overrightarrow{OB}$ to $\overrightarrow{AD}$. A few candidates also achieved the correct result by arguing that the midpoints of the two diagonals of a parallelogram are coincident. Occasionally the incorrect answer of $\overrightarrow{OC}$ = ±(4$\mathbf{i}$ + 2$\mathbf{j}$ + $\mathbf{k}$) was given, which is a result of taking the difference between $\overrightarrow{OD}$ and $\overrightarrow{AB}$.

Candidates who were successful in part (e) found the area of the parallelogram either by finding the area of triangle ABD using $\frac{1}{2}bd$ sin A and doubling the result or by applying a method of base × perpendicular height. The most common error in part (e) was for candidates to find the product of lengths AD and AB.

Candidates who were successful in part (f) usually found the shortest distance by multiplying their AD by sin 71° (or equivalent). Those candidates who multiplied AB by sin 71° did not receive any credit. A few candidates attempted to use vectors to find $\left|\overrightarrow{DE}\right|$, where E is the point where the perpendicular from D meets the line l, often spending considerable time for usually little or no reward.

Question 8

In general, this was the most poorly answered question on the paper with about 15% of candidates who failed to score and about 11% of candidates gaining 1 mark usually in part (a). This question discriminated well between candidates of higher abilities, with about 27% of candidates gaining at least 8 of the 12 marks available and only about 7% of candidates gaining all 12 marks. Many weaker candidates made little or no progress in part (b), maybe because of the generalised nature of the differential equation.

In part (a), a significant number of candidates were not clear or precise in their explanations. A number of them used the word “mass” and it was not clear whether they were referring to the mass of the unburned fuel or the mass of the waste products.

In part (b), those candidates who were able to separate the variables, were usually able to integrate both sides correctly, although a number of candidates integrated $\dfrac{1}{M-x}$ incorrectly to give ln $(M - x)$. Many others substituted $t = 0$, $x = 0$ immediately after integration, to find their constant of integration as $-\ln M$ and most used a variety of correct methods to eliminate logarithms in order to find $x = M(1 - e^{-kt})$ (or equivalent). A significant number of candidates, however, correctly rearranged their integrated expression into the form $x = M - Ae^{-kt}$ before using $t = 0$, $x = 0$ to correctly find A. Common errors in this part included omitting the constant of integration or treating M as a variable. Also, a number of candidates struggled to remove logarithms correctly and gave an equation of the form $M - x = e^{-kt} + e^{c}$ which was then sometimes manipulated to $M - x = Ae^{-kt}$.

In part (c), some candidates were able to substitute $t = \ln 4$, $x = \frac{1}{2}M$ into one of their equations involving x and t, but only a minority were able to find a numeric value of k. Only the most able candidates were able to find $k = \frac{1}{2}$ and substitute this into their equation together with $t = \ln 9$ to find $x = \frac{2}{3}M$.

Statistics for C4 Practice Paper Silver Level S2

				Mean score for students achieving grade:							
Qu	**Max score**	**Modal score**	**Mean %**	**ALL**	**A***	**A**	**B**	**C**	**D**	**E**	**U**
1	4		84	3.34	3.91	3.70	3.45	3.17	2.79	2.36	1.66
2	6		71	4.28	5.83	5.13	4.10	3.12	2.24	1.46	0.51
3	7		74	5.20	6.72	6.02	5.43	4.70	3.95	2.91	1.45
4	15		67	9.99	14.23	12.43	10.19	7.93	5.88	4.41	2.94
5	11		68	7.49	10.31	8.79	7.52	6.39	5.34	4.19	2.57
6	5		65	3.23		4.34	3.12	2.26	1.59	0.86	0.56
7	15		60	9.04	13.94	10.95	7.98	6.17	4.54	3.55	1.61
8	12		41	4.93	9.74	5.48	3.21	1.63	1.20	0.95	0.33
	75		**63**	**47.50**		**56.84**	**45.00**	**35.37**	**27.53**	**20.69**	**11.63**